Solutions - Homework 1

(Due date: January 19th @ 11:59 pm)

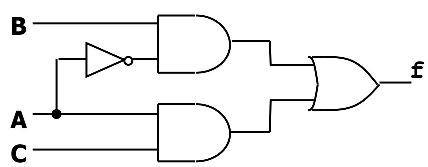
Presentation and clarity are very important! Show your procedure!

PROBLEM 1 (28 PTS)

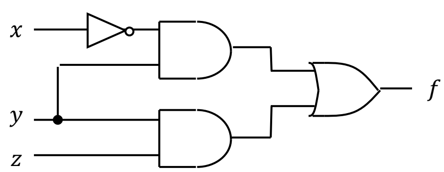
a) Simplify the following functions using ONLY Boolean Algebra Theorems. For each resulting simplified function, sketch the logic circuit using AND, OR, XOR, and NOT gates. (15 pts)

✓ $F(a,b,c) = \prod(M_0, M_1, M_4, M_6)$ ✓ $F = \overline{x(y \oplus z) + \bar{y}}$ ✓ $F = (A + \bar{B} + D)(\bar{A}B + \bar{D})$

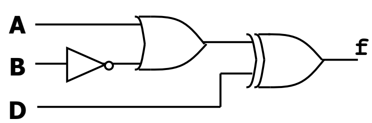
✓ $F(A,B,C) = \prod(M_0, M_1, M_4, M_6) = \sum(m_2, m_3, m_5, m_7) = \bar{A}B\bar{C} + \bar{A}BC + A\bar{B}C + ABC = \bar{A}B(\bar{C} + C) + AC(\bar{B} + B)$
$= \bar{A}B + AC$



✓ $F = \overline{x(y \oplus z) + \bar{y}} = \overline{x(y \oplus z)}.\, y = (\bar{x} + \overline{y \oplus z})y = (\bar{x} + yz + \bar{y}\bar{z})y = \bar{x}y + yz$



✓ $F = (A + \bar{B} + D)(\bar{A}B + \bar{D}) = (X + D)(\bar{X} + \bar{D}) = X\bar{D} + \bar{X}D, X = A + \bar{B}$
$= (A + \bar{B})\bar{D} + \bar{A}BD = A\bar{D} + \bar{B}\bar{D} + \bar{A}BD = \bar{D}(A + \bar{B}) + D\bar{A}B$



b) Determine whether or not the following expression is valid, i.e., whether the left- and right-hand sides represent the same function. Suggestion: complete the truth tables for both sides: (5 pts)

$$x_1x_3 + \overline{x_2}\,\overline{x_3} + \overline{x_1}x_2 = x_2x_3 + \overline{x_1}\,\overline{x_3} + x_1\,\overline{x_2}$$

Left-hand side:

$x_1(x_2 + \overline{x_2})\,x_3 + (x_1 + \overline{x_1})\overline{x_2}\,\overline{x_3} + \overline{x_1}x_2(x_3 + \overline{x_3}) = x_1x_2x_3 + x_1\,\overline{x_2}\,x_3 + x_1\overline{x_2}\,\overline{x_3} + \overline{x_1}\,\overline{x_2}\,\overline{x_3} + \overline{x_1}\,x_2x_3 + \overline{x_1}x_2\overline{x_3}$
$= \sum m(0,2,3,4,5,7)$

Right-hand side:

$(x_1 + \overline{x_1})x_2x_3 + \overline{x_1}(x_2 + \overline{x_2})\,\overline{x_3} + x_1\,\overline{x_2}(x_3 + \overline{x_3}) = x_1x_2x_3 + \overline{x_1}x_2x_3 + \overline{x_1}x_2\,\overline{x_3} + \overline{x_1}\,\overline{x_2}\,\overline{x_3} + x_1\,\overline{x_2}x_3 + x_1\,\overline{x_2}\,\overline{x_3}$
$= \sum m(0,2,3,4,5,7)$

Both left-hand and right-hand equations represent the same Boolean function.

c) For the following Truth table with two outputs: (8 pts)

- Provide the Boolean functions using the Canonical Sum of Products (SOP), and Product of Sums (POS). (4 pts)
- Express the Boolean functions using the minterms and maxterms representations.
- Sketch the logic circuits as Canonical Sum of Products and Product of Sums. (3 pts)

x	y	z	f_1	f_2
0	0	0	1	0
0	0	1	0	1
0	1	0	0	1
0	1	1	0	0
1	0	0	0	1
1	0	1	1	0
1	1	0	0	0
1	1	1	1	0

Sum of Products

$f_1 = \bar{x}\bar{y}\bar{z} + x\bar{y}z + xyz$

$f_2 = \bar{x}\bar{y}z + \bar{x}y\bar{z} + x\bar{y}\bar{z}$

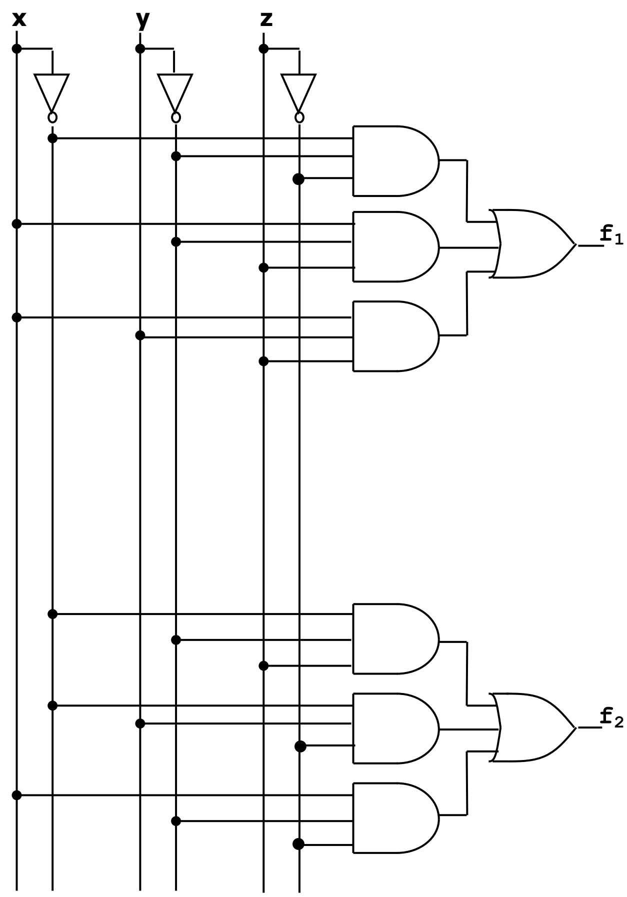
Product of Sums

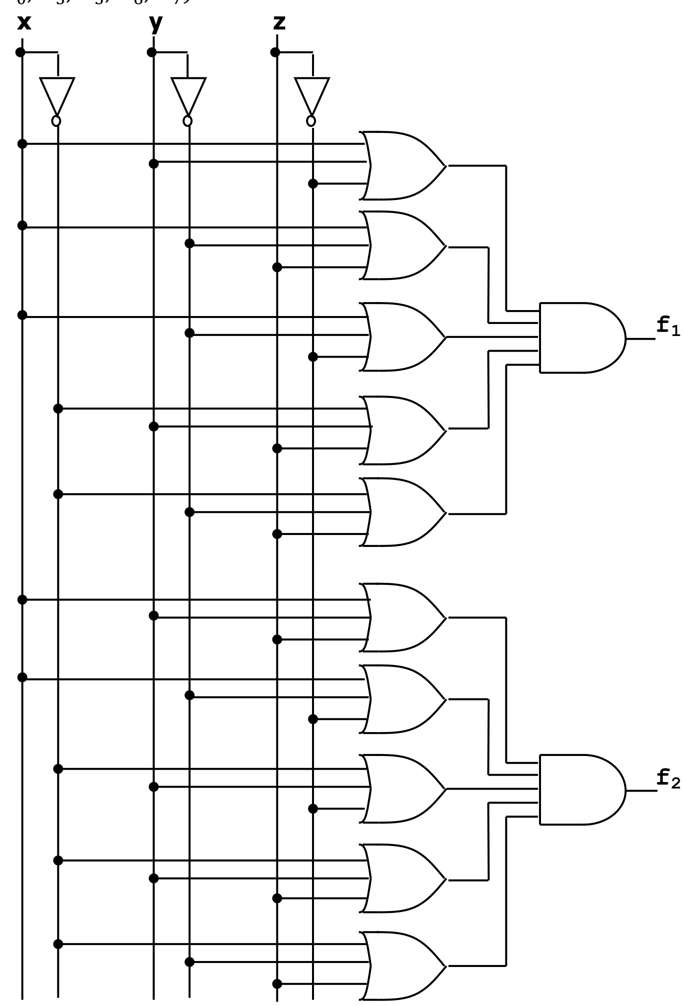
$f_1 = (x + y + \bar{z})(x + \bar{y} + z)(x + \bar{y} + \bar{z})(\bar{x} + y + z)(\bar{x} + \bar{y} + z)$

$f_2 = (x + y + z)(x + \bar{y} + \bar{z})(\bar{x} + y + \bar{z})(\bar{x} + \bar{y} + z)(\bar{x} + \bar{y} + \bar{z})$

Minterms and maxterms: $f_1 = \sum(m_0, m_5, m_7) = \prod(M_1, M_2, M_3, M_4, M_6)$.

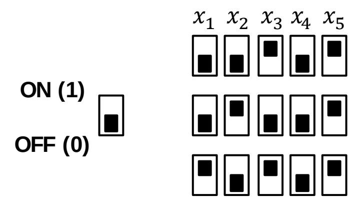
$f_2 = \sum(m_1, m_2, m_4) = \prod(M_0, M_3, M_5, M_6, M_7)$.





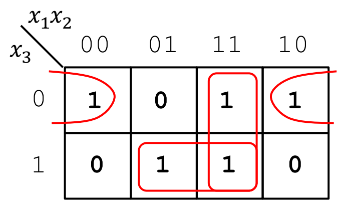
PROBLEM 2 (18 PTS)

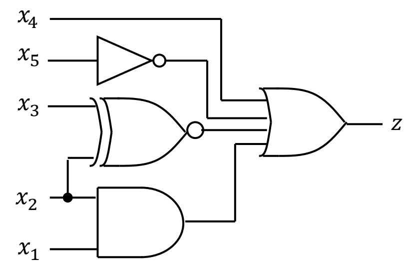
a) Security combinations: A lock only opens ($z = 0$) when the 5 switches (x_1, x_2, x_3, x_4, x_5) are set in any of the 3 configurations shown in the figure, otherwise the lock is closed ($z = 1$). A switch generates a '1' in the ON position, and a '0' in the OFF position.



- Provide the simplified Boolean equation for the output z and sketch the logic circuit.

x_1	x_2	x_3	x_4	x_5	z
0	0	1	0	1	0
0	1	0	0	1	0
1	0	1	0	1	0
All remaining cases					1





$\bar{z} = \bar{x}_1 \bar{x}_2 x_3 \bar{x}_4 x_5 + \bar{x}_1 x_2 \bar{x}_3 \bar{x}_4 x_5 + x_1 \bar{x}_2 x_3 \bar{x}_4 x_5 = \bar{x}_4 x_5 (\bar{x}_1 \bar{x}_2 x_3 + \bar{x}_1 x_2 \bar{x}_3 + x_1 \bar{x}_2 x_3)$

$\bar{z} = \bar{x}_4 x_5 . f(x_1, x_2, x_3)$

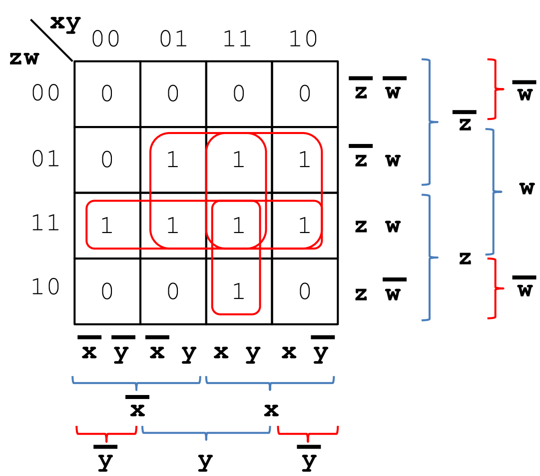
* $f(x_1, x_2, x_3) = \sum m(1,2,5) \rightarrow \bar{f}(x_1, x_2, x_3) = \sum m(0,3,4,6,7)$

$z = \overline{\bar{x}_4 x_5 . f(x_1, x_2, x_3)} = \overline{\bar{x}_4 x_5} + \bar{f}(x_1, x_2, x_3) = x_4 + \bar{x}_5 + x_2 x_3 + \bar{x}_2 \bar{x}_3 + x_1 x_2$

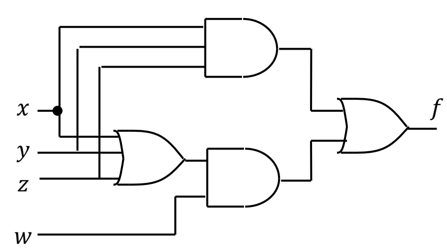
b) A doctoral student is defending his Dissertation. A 4-member committee determines whether to accept or reject the work. A simple majority vote is required. In case of a tie, the outcome is determined by the vote of the chair of the committee.

- Design the circuit (provide the simplified Boolean equation and sketch the logic circuit) that generates $f = 1$ if the committee accepts the work, and $f = 0$ if the work is rejected. We assign x, y, z, w to the vote of each committee member (w is the vote of the chair of the committee), where '1' means accept, and '0' reject. (8 pts)

x	y	z	w	f
0	0	0	0	0
0	0	0	1	0
0	0	1	0	0
0	0	1	1	1
0	1	0	0	0
0	1	0	1	1
0	1	1	0	0
0	1	1	1	1
1	0	0	0	0
1	0	0	1	1
1	0	1	0	0
1	0	1	1	1
1	1	0	0	0
1	1	0	1	1
1	1	1	0	1
1	1	1	1	1

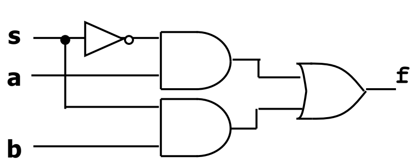


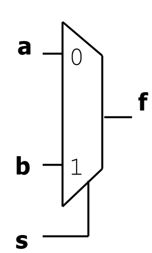
$f = yw + xw + zw + xyz$



PROBLEM 3 (11 PTS)

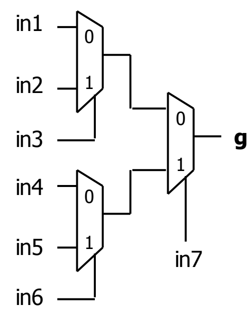
a) The following circuit has the following logic function: $f = \bar{s}a + sb$.
✓ Complete the truth table of the circuit, and sketch the logic circuit (3 pts)





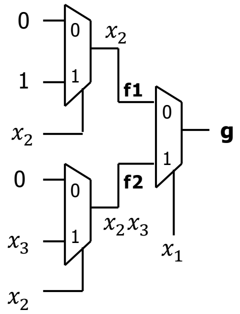
s	a	b	f
0	0	0	0
0	0	1	0
0	1	0	1
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	0
1	1	1	1

b) We can use several instances of the previous circuit to implement different functions. (8 pts)



- For example, the following selection of inputs produce the function: $g = \overline{x_1}x_2 + x_2x_3$. Demonstrate that this is the case.

in1	in2	in3	in4	in5	in6	in7
0	1	x_2	0	x_3	x_2	x_1

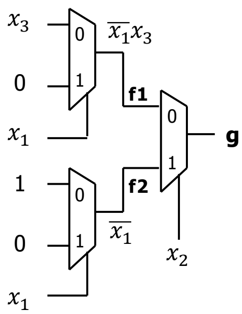


$f_1 = \overline{x_2}(0) + x_2(1) = x_2$
$f_2 = \overline{x_2}(0) + x_2(x_3) = x_2x_3$

$g = \overline{x_1}(x_2) + x_1(x_2x_3) = x_2(\overline{x_1} + x_1x_3) = x_2(\overline{x_1} + x_1)(\overline{x_1} + x_3)$
$g = x_2(\overline{x_1} + x_3) = \overline{x_1}x_2 + x_2x_3$

- Given the following inputs, provide the resulting function g (minimize the function).

in1	in2	in3	in4	in5	in6	in7
x_3	0	x_1	1	0	x_1	x_2



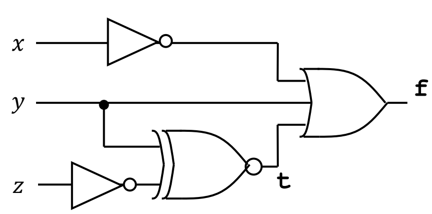
$f_1 = \overline{x_1}(x_3) + x_1(0) = \overline{x_1}(x_3)$
$f_2 = \overline{x_1}(1) + x_3(0) = \overline{x_1}$

$g = \overline{x_2}(\overline{x_1}x_3) + x_2(\overline{x_1}) = \overline{x_1}(\overline{x_2}x_3 + x_2) = \overline{x_1}(x_2 + \overline{x_2})(x_2 + x_3)$

$g = \overline{x_1}x_2 + \overline{x_1}x_3$

PROBLEM 4 (25 PTS)

a) Complete the truth table describing the output of the following circuit and write the simplified Boolean equation (6 pts).

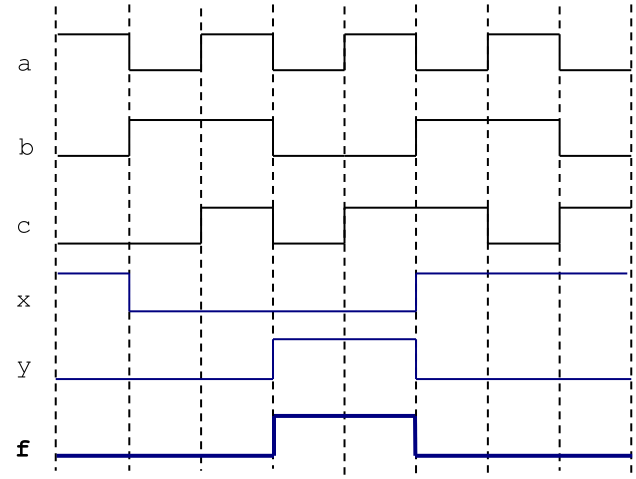


x	y	z	t	f
0	0	0	0	1
0	0	1	1	1
0	1	0	1	1
0	1	1	0	1
1	0	0	0	0
1	0	1	1	1
1	1	0	1	1
1	1	1	0	1

$f = \bar{x} + y + (y \oplus z) = \bar{x} + y + z$

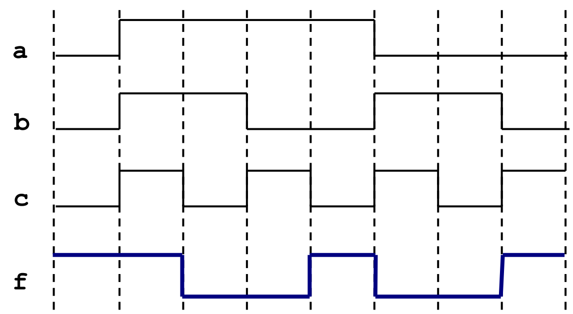
b) Complete the timing diagram of the logic circuit whose VHDL description is shown below: (6 pts)

```
library ieee;
use ieee.std_logic_1164.all;

entity circ is
  port ( a, b, c: in std_logic;
          f: out std_logic);
end circ;

architecture struct of circ is
  signal x, y: std_logic;

begin
  x <= not(a) xor not(c);
  f <= y and (not b);
  y <= x nor b;
end struct;
```



c) The following is the timing diagram of a logic circuit with 3 inputs. Sketch the logic circuit that generates this waveform. Then, complete the VHDL code (using VHDL signals is optional). (8 pts)

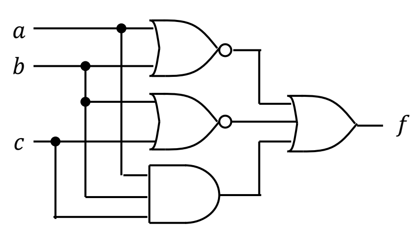
```
library ieee;
use ieee.std_logic_1164.all;

entity wave is
  port ( a, b, c: in std_logic;
              f: out std_logic);
end wave;

architecture struct of wave is
   signal x: std_logic;
begin
   x <= (a and b and c) or (a nor b);
   f <= (b nor c) or x;
end struct;
```



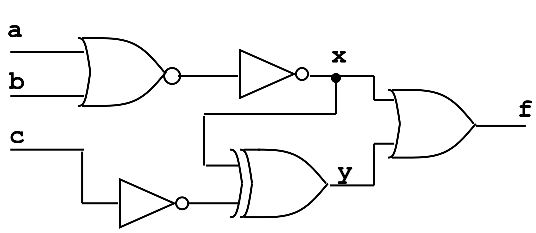
a	b	c	f
0	0	0	1
0	0	1	1
0	1	0	0
0	1	1	0
1	0	0	1
1	0	1	0
1	1	0	0
1	1	1	1

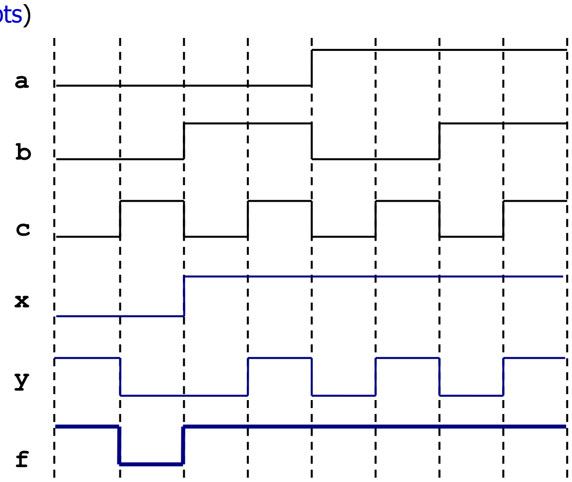
c \ ab	00	01	11	10
0	1	0	0	1
1	1	0	1	0

$f = \bar{b}\bar{c} + \bar{a}\bar{b} + abc$



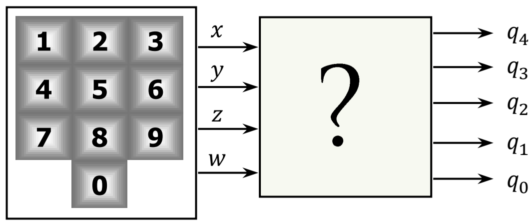
d) Complete the timing diagram of the following circuit: (5 pts)

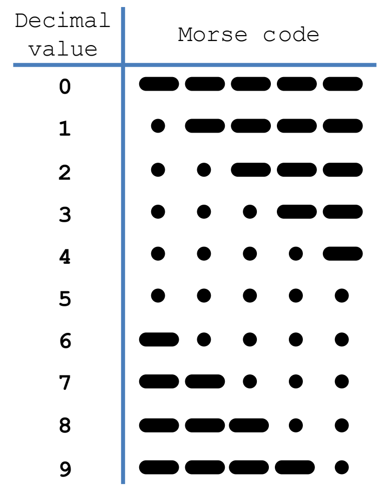




PROBLEM 5 (18 PTS)

- A numeric keypad produces a 4-bit code $xyzw$ representing an unsigned number from 0 to 9. We want to design a logic circuit that converts each 4-bit code to Morse code (where alphanumeric characters are encoded into sequences of dots and dashes). The figure depicts the Morse code representations for numbers from 0 to 9. The circuit generates 5 bits, where a '0' represents a dot, and '1' represents a dash.





✓ Complete the truth table for each output (q_4, q_3, q_2, q_1, q_0). (3 pts)

✓ Provide the simplified expression for each output (q_4, q_3, q_2, q_1, q_0). Use Karnaugh maps for q_4, q_3, q_2, and the Quine-McCluskey algorithm for q_1, q_0. Note it is safe to assume that the codes `1010` to `1111` will not be produced by the keypad. (15 pts)

Value	x	y	z	w	q_4	q_3	q_2	q_1	q_0
0	0	0	0	0	1	1	1	1	1
1	0	0	0	1	0	1	1	1	1
2	0	0	1	0	0	0	1	1	1
3	0	0	1	1	0	0	0	1	1
4	0	1	0	0	0	0	0	0	1
5	0	1	0	1	0	0	0	0	0
6	0	1	1	0	1	0	0	0	0
7	0	1	1	1	1	1	0	0	0
8	1	0	0	0	1	1	1	0	0
9	1	0	0	1	1	1	1	1	0
	1	0	1	0	X	X	X	X	X
	1	0	1	1	X	X	X	X	X
	1	1	0	0	X	X	X	X	X
	1	1	0	1	X	X	X	X	X
	1	1	1	0	X	X	X	X	X
	1	1	1	1	X	X	X	X	X

q_4

zw \ xy	00	01	11	10
00	1	0	X	1
01	0	0	X	1
11	0	1	X	X
10	0	1	X	X

q_3

zw \ xy	00	01	11	10
00	1	0	X	1
01	1	0	X	1
11	0	1	X	X
10	0	0	X	X

q_2

zw \ xy	00	01	11	10
00	1	0	X	1
01	1	0	X	1
11	0	0	X	X
10	1	0	X	X

$q_4 = \bar{y}\bar{z}\bar{w} + x\bar{y} + yz$

$q_3 = \bar{y}\bar{z} + x\bar{y} + yzw$

$q_2 = \bar{y}\bar{z} + \bar{w}\bar{y} + x$

- $q_1 = \sum m(0,1,2,3,9) + \sum d(10,11,12,13,14,15)$.

Number of ones	4-literal implicants	3-literal implicants	2-literal implicants
0	m_0 = 0000 ✓	$m_{0,1}$ = 000- ✓ $m_{0,2}$ = 00-0 ✓	**$m_{0,1,2,3}$ = 00--** ~~$m_{0,2,1,3}$ = 00--~~
1	m_1 = 0001 ✓ m_2 = 0010 ✓	$m_{1,3}$ = 00-1 ✓ $m_{1,9}$ = -001 ✓ $m_{2,3}$ = 001- ✓ $m_{2,10}$ = -010 ✓	**$m_{1,3,9,11}$ = -0-1** ~~$m_{1,9,3,11}$ = -0-1~~ **$m_{2,3,10,11}$ = -01-** ~~$m_{2,10,3,11}$ = -01-~~
2	m_3 = 0011 ✓ m_9 = 1001 ✓ m_{10}= 1010 ✓ m_{12}= 1100 ✓	$m_{3,11}$ = -011 ✓ $m_{9,11}$ = 10-1 ✓ $m_{9,13}$ = 1-01 ✓ $m_{10,11}$ = 101- ✓ $m_{10,14}$ = 1-10 ✓ $m_{12,13}$ = 110- ✓ $m_{12,14}$ = 11-0 ✓	**$m_{9,11,13,15}$ = 1--1** ~~$m_{9,13,11,15}$ = 1--1~~ **$m_{10,11,14,15}$ = 1-1-** ~~$m_{10,14,11,15}$ = 1-1-~~ **$m_{12,13,14,15}$ = 11--** ~~$m_{12,14,13,15}$ = 11--~~
3	m_{11}= 1011 ✓ m_{13}= 1101 ✓ m_{14}= 1110 ✓	$m_{11,15}$ = 1-11 ✓ $m_{13,15}$ = 11-1 ✓ $m_{14,15}$ = 111- ✓	
4	m_{15}= 1111 ✓		

$q_1 = \bar{x}\bar{y} + \bar{y}w + \bar{y}z + xw + xz + xy$

Prime Implicants		Minterms 0	1	2	3	9
$m_{0,1,2,3}$	$\bar{x}\bar{y}$	**X**	X	X	X	
$m_{1,3,9,11}$	$\bar{y}w$		X		X	X
$m_{2,3,10,11}$	$\bar{y}z$			X	X	
$m_{9,11,13,15}$	xw					**X**
$m_{10,11,14,15}$	xz					
$m_{12,13,14,15}$	xy					

$\Rightarrow q_1 = \bar{x}\bar{y} + xw$

- $q_0 = \sum m(0,1,2,3,4) + \sum d(10,11,12,13,14,15)$.

Number of ones	4-literal implicants	3-literal implicants	2-literal implicants
0	m_0 = 0000 ✓	$m_{0,1}$ = 000- ✓ $m_{0,2}$ = 00-0 ✓ **$m_{0,4}$ = 0-00**	**$m_{0,1,2,3}$ = 00--** ~~$m_{0,2,1,3}$ = 00--~~
1	m_1 = 0001 ✓ m_2 = 0010 ✓ m_4 = 0100 ✓	$m_{1,3}$ = 00-1 ✓ $m_{2,3}$ = 001- ✓ $m_{2,10}$ = -010 ✓ **$m_{4,12}$ = -100**	**$m_{2,3,10,11}$ = -01-** ~~$m_{2,10,3,11}$ = -01-~~
2	m_3 = 0011 ✓ m_{10}= 1010 ✓ m_{12}= 1100 ✓	$m_{3,11}$ = -011 ✓ $m_{10,11}$ = 101- ✓ $m_{10,14}$ = 1-10 ✓ $m_{12,13}$ = 110- ✓ $m_{12,14}$ = 11-0 ✓	**$m_{10,11,14,15}$ = 1-1-** ~~$m_{10,14,11,15}$ = 1-1-~~ **$m_{12,13,14,15}$ = 11--** ~~$m_{12,14,13,15}$ = 11--~~
3	m_{11}= 1011 ✓ m_{13}= 1101 ✓ m_{14}= 1110 ✓	$m_{11,15}$ = 1-11 ✓ $m_{13,15}$ = 11-1 ✓ $m_{14,15}$ = 111- ✓	
4	m_{15}= 1111 ✓		

$q_0 = \bar{x}\bar{z}\bar{w} + \bar{x}\bar{y} + \bar{y}z + xz + xy$

Prime Implicants		Minterms 0	1	2	3	4
$m_{0,4}$	$\bar{x}\bar{z}\bar{w}$	**X**				X
$m_{4,12}$	$y\bar{z}\bar{w}$					X
$m_{0,1,2,3}$	$\bar{x}\bar{y}$	**X**	X	X	X	
$m_{2,3,10,11}$	$\bar{y}z$			X	X	
$m_{10,11,14,15}$	xz					
$m_{12,13,14,15}$	xy					

$\Rightarrow q_0 = \bar{x}\bar{y} + \bar{x}\bar{z}\bar{w}$